ME 141

Engineering Mechanics

Lecture 13: Kinematics of rigid bodies

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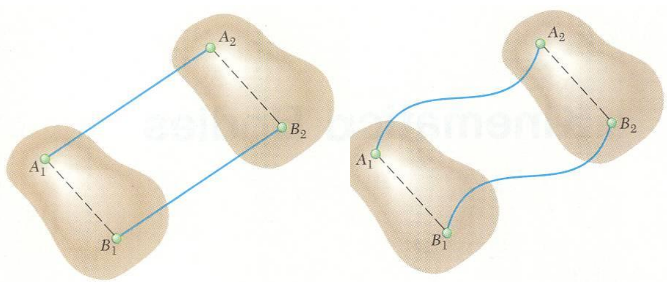
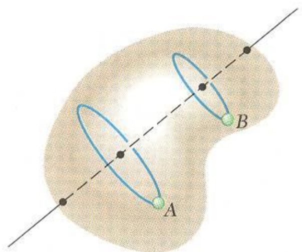
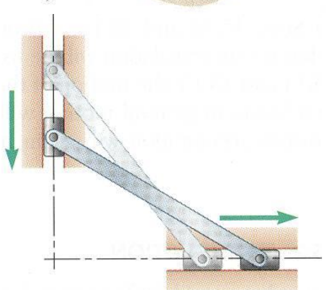
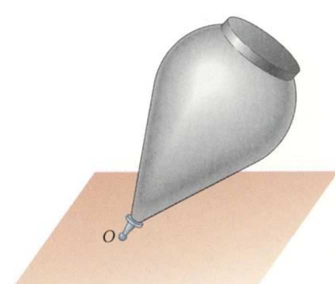
Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Introduction



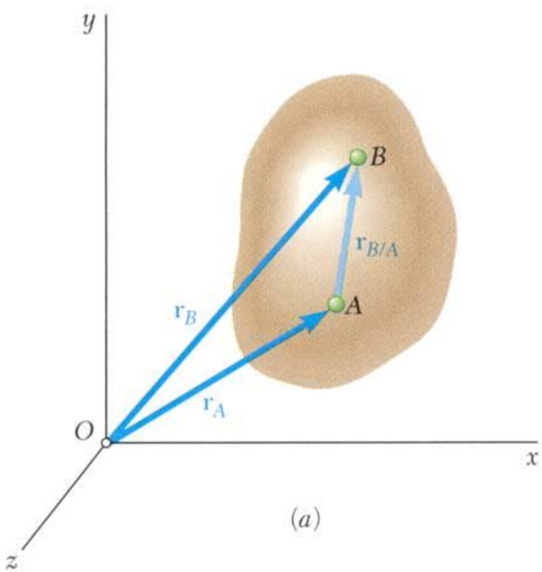


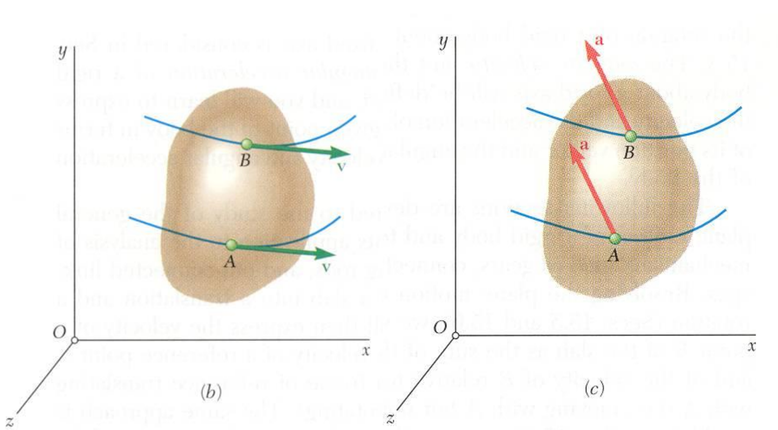




- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.
- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

Translation





- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.
- For any two particles in the body,

  $$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

  $$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

  $$\vec{v}_B = \vec{v}_A$$

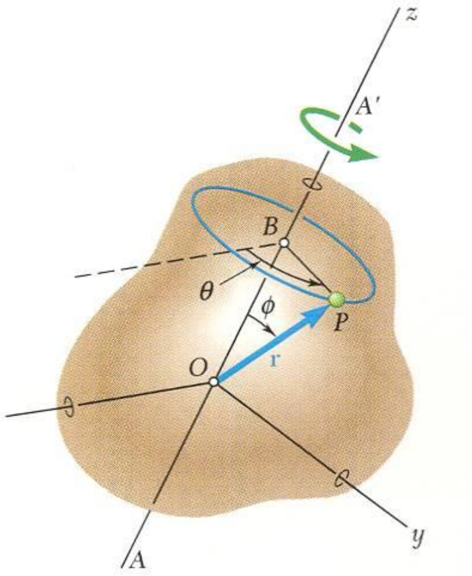
  All particles have the same velocity.
- Differentiating with respect to time again,

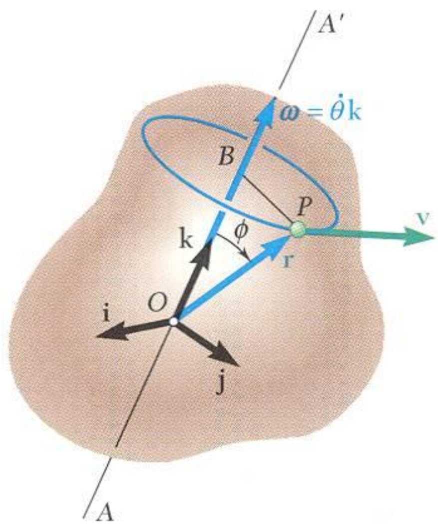
  $$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

  $$\vec{a}_B = \vec{a}_A$$

  All particles have the same acceleration.

Rotation About a Fixed Axis. Velocity





- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

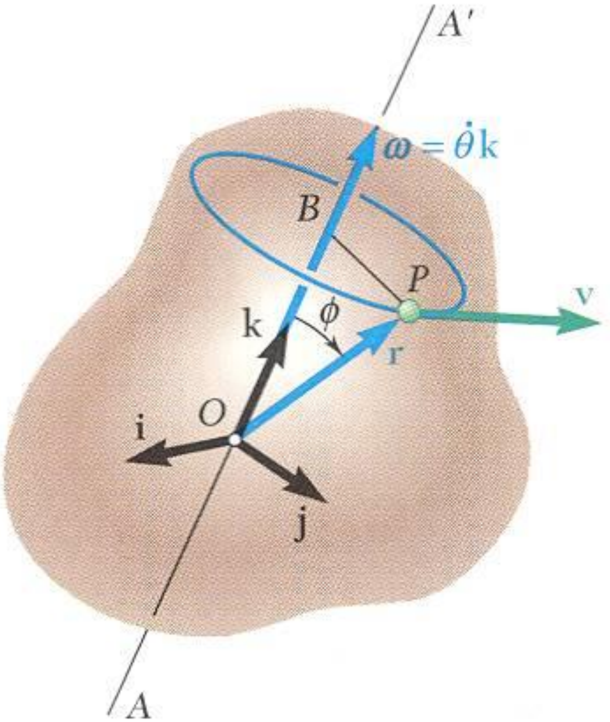
$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \textit{angular velocity}$$

Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega}\times\vec{r})$$

$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$

- $\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$

$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

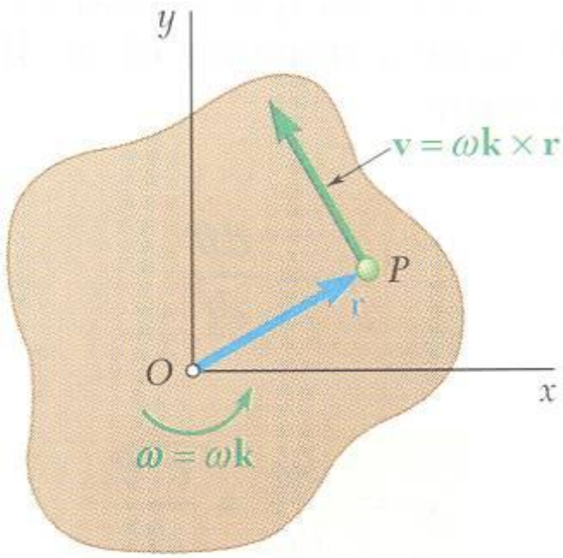
- Acceleration of P is combination of two vectors,

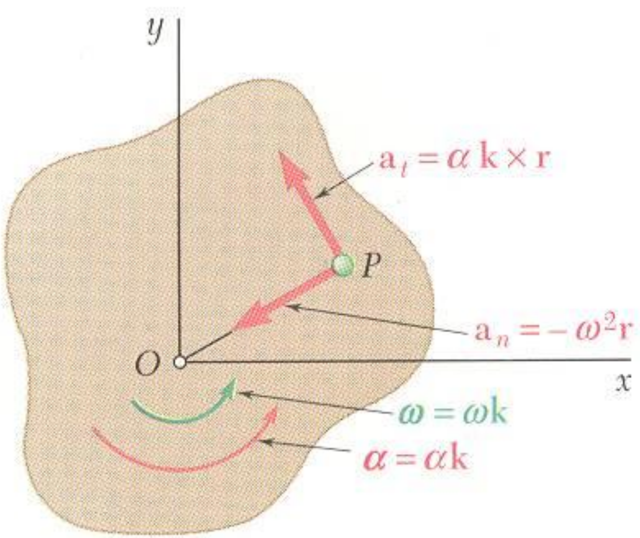
$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

$$\vec{\alpha}\times\vec{r} = \text{tangential acceleration component}$$

$$\vec{\omega}\times\vec{\omega}\times\vec{r} = \text{radial acceleration component}$$

Rotation About a Fixed Axis. Representative





- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point *P* of the slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega\vec{k} \times \vec{r}$$

$$v = r\omega$$

- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$

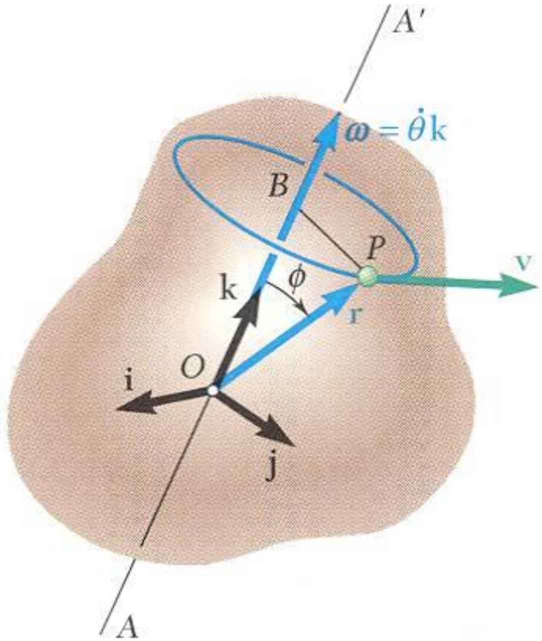
$$= \alpha\vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha\vec{k} \times \vec{r} \qquad a_t = r\alpha$$

$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \dfrac{d\theta}{dt}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$$

- *Uniform Rotation*, $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

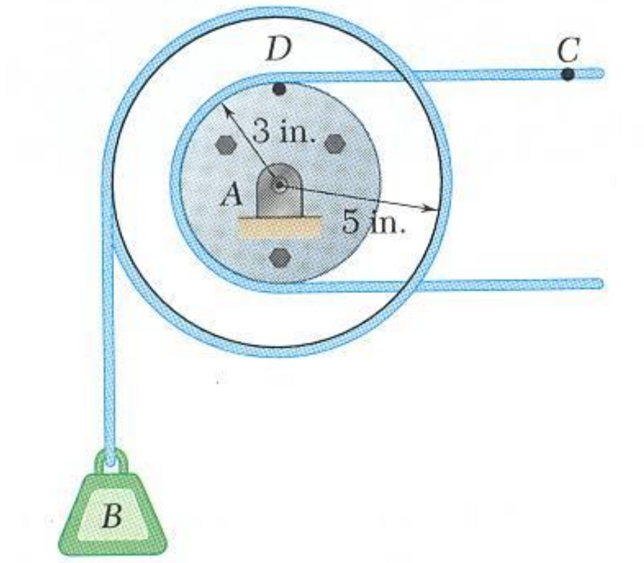
- *Uniformly Accelerated Rotation*, α = constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Sample Problem 5.1



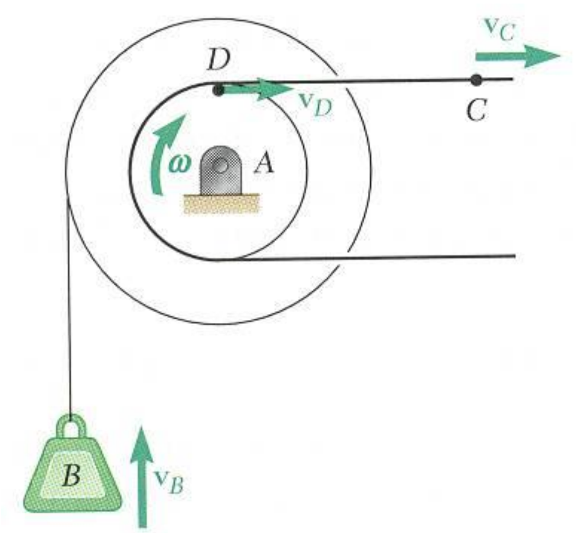
Cable *C* has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right.

Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load *B* after 2 s, and *(c)* the acceleration of the point *D* on the rim of the inner pulley at $t = 0$.

<u>SOLUTION</u>:

- Due to the action of the cable, the tangential velocity and acceleration of *D* are equal to the velocity and acceleration of *C*. Calculate the initial angular velocity and acceleration.
- Apply the relations for uniformly accelerated rotation to determine the velocity and angular position of the pulley after 2 s.
- Evaluate the initial tangential and normal acceleration components of *D*.

Sample Problem 5.1



SOLUTION:

- The tangential velocity and acceleration of D are equal to the velocity and acceleration of C.

$$(\vec{v}_D)_0 = (\vec{v}_C)_0 = 12\,\text{in./s} \rightarrow$$

$$(v_D)_0 = r\omega_0$$

$$\omega_0 = \frac{(v_D)_0}{r} = \frac{12}{3} = 4\,\text{rad/s}$$

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_t = r\alpha$$

$$\alpha = \frac{(a_D)_t}{r} = \frac{9}{3} = 3\,\text{rad/s}^2$$

- Apply the relations for uniformly accelerated rotation to determine velocity and angular position of pulley after 2 s.

$$\omega = \omega_0 + \alpha t = 4\,\text{rad/s} + \left(3\,\text{rad/s}^2\right)(2\,\text{s}) = 10\,\text{rad/s}$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4\,\text{rad/s})(2\,\text{s}) + \tfrac{1}{2}\left(3\,\text{rad/s}^2\right)(2\,\text{s})^2$$

$$= 14\,\text{rad}$$

$$N = (14\,\text{rad})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \text{number of revs}$$

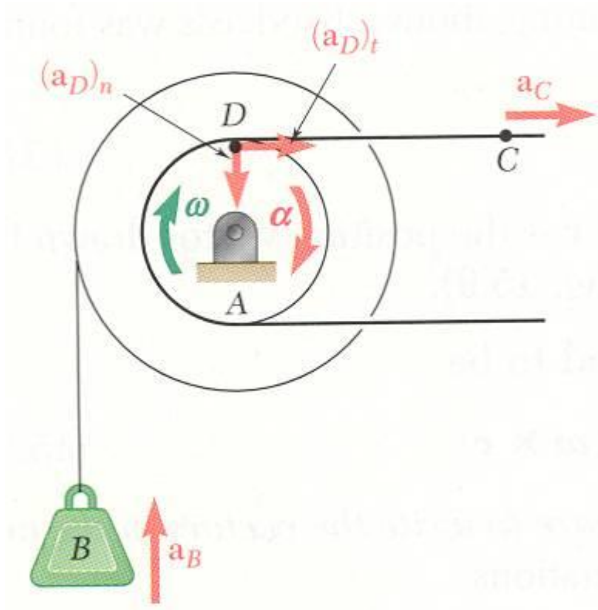
$$\boxed{N = 2.23\,\text{rev}}$$

$$v_B = r\omega = (5\,\text{in.})(10\,\text{rad/s})$$

$$\Delta y_B = r\theta = (5\,\text{in.})(14\,\text{rad})$$

$$\boxed{\vec{v}_B = 50\,\text{in./s} \uparrow \quad \Delta y_B = 70\,\text{in.}}$$

Sample Problem 5.1

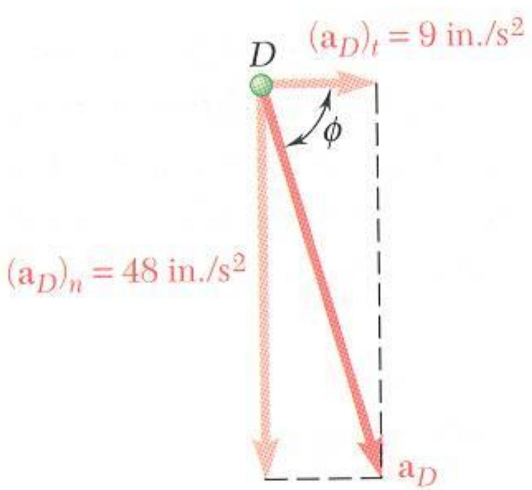


- Evaluate the initial tangential and normal acceleration components of *D*.

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_n = r_D\omega_0^2 = (3\,\text{in.})(4\,\text{rad/s})^2 = 48\,\text{in/s}^2$$

$$(\vec{a}_D)_t = 9\,\text{in./s}^2 \rightarrow \qquad (\vec{a}_D)_n = 48\,\text{in./s}^2 \downarrow$$



Magnitude and direction of the total acceleration,

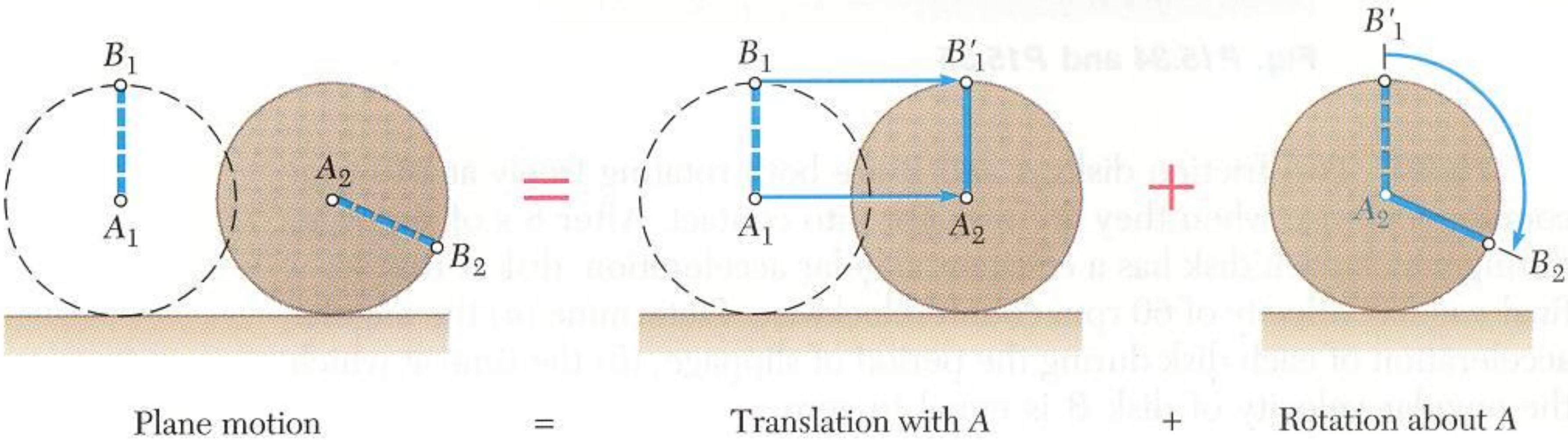
$$a_D = \sqrt{(a_D)_t^2 + (a_D)_n^2}$$
$$= \sqrt{9^2 + 48^2}$$

$$a_D = 48.8\,\text{in./s}^2$$

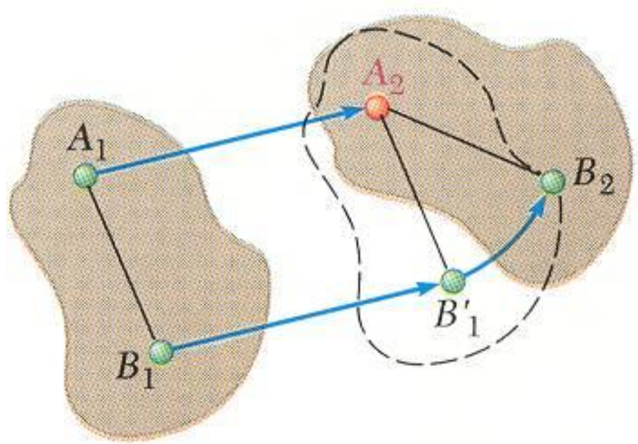
$$\tan\phi = \frac{(a_D)_n}{(a_D)_t}$$
$$= \frac{48}{9}$$

$$\phi = 79.4°$$

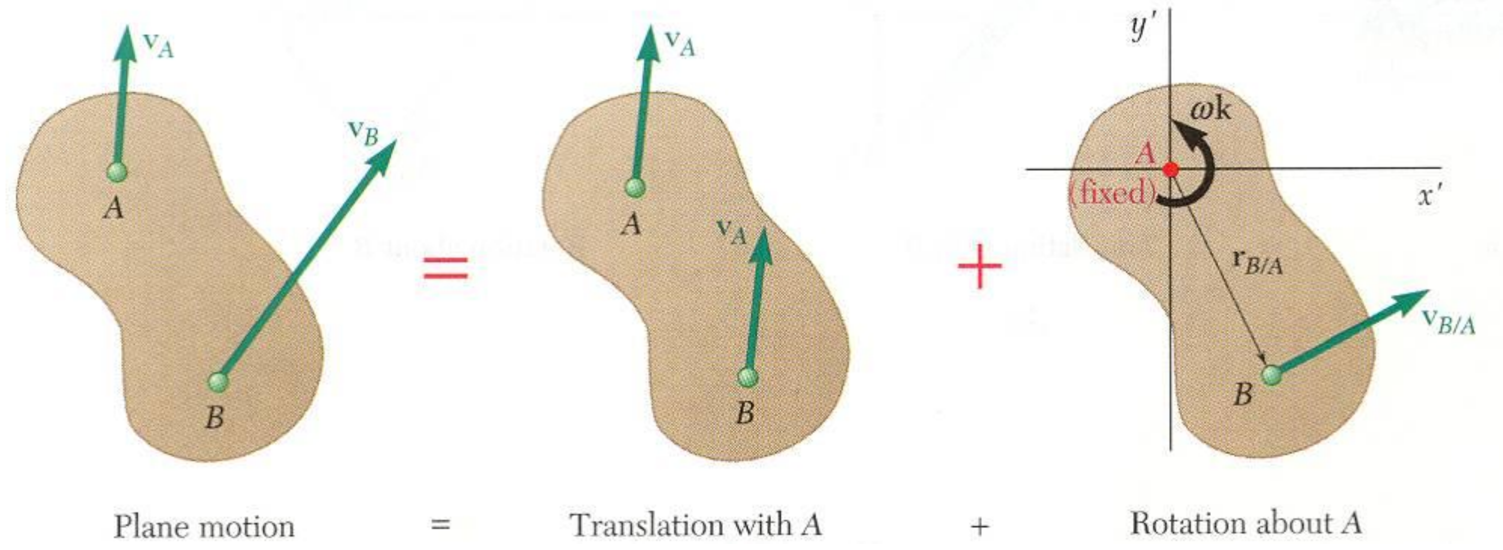
General Plane Motion

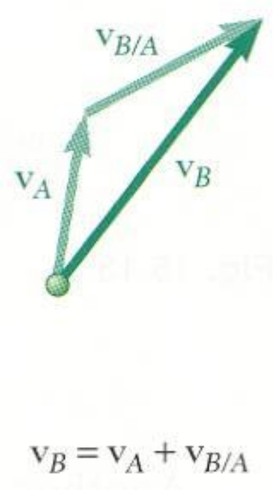


- *General plane motion* is neither a translation nor a rotation.
- General plane motion can be considered as the *sum* of a translation and rotation.
- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B'_1
  - rotation of B'_1 about A_2 to B_2



Absolute and Relative Velocity in Plane Motion





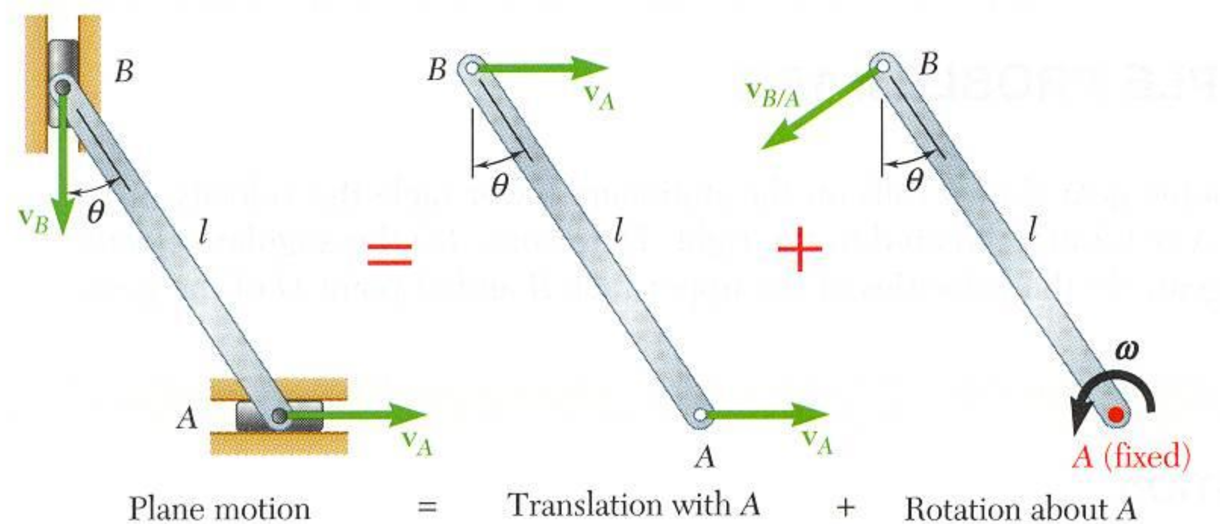
- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

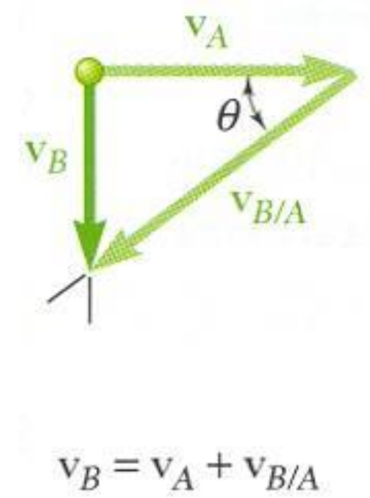
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

Absolute and Relative Velocity in Plane





- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.
- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

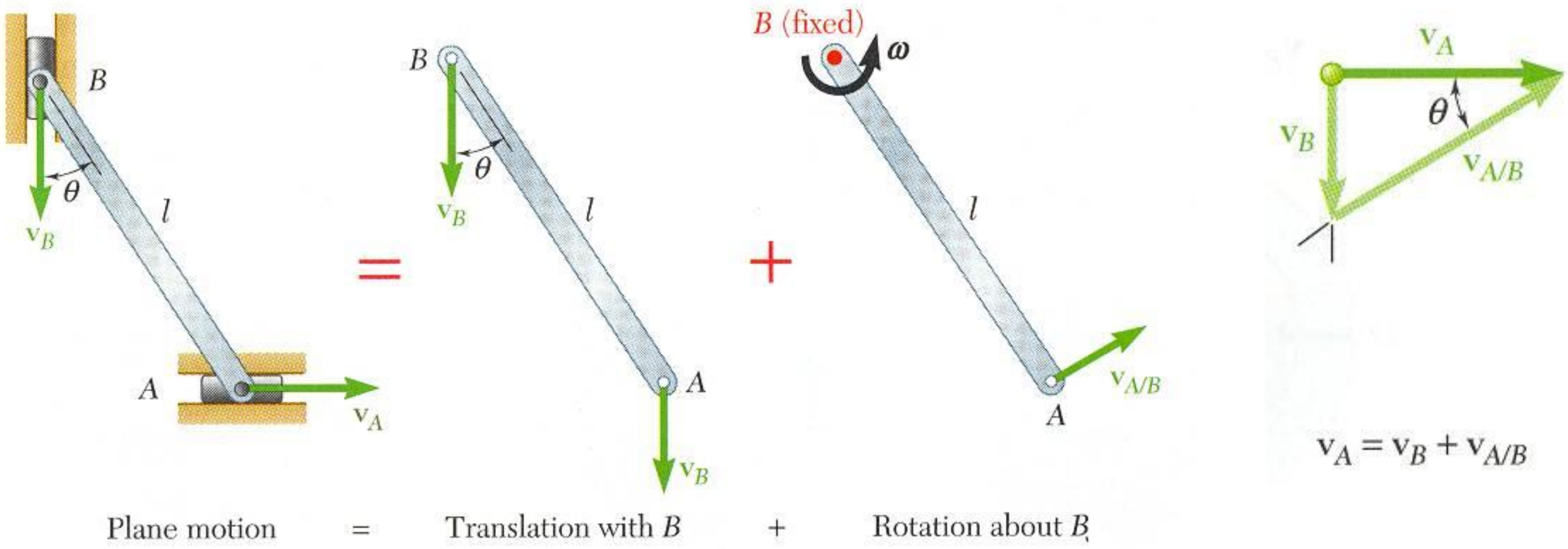
$$\frac{v_B}{v_A} = \tan\theta$$

$$v_B = v_A \tan\theta$$

$$\frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

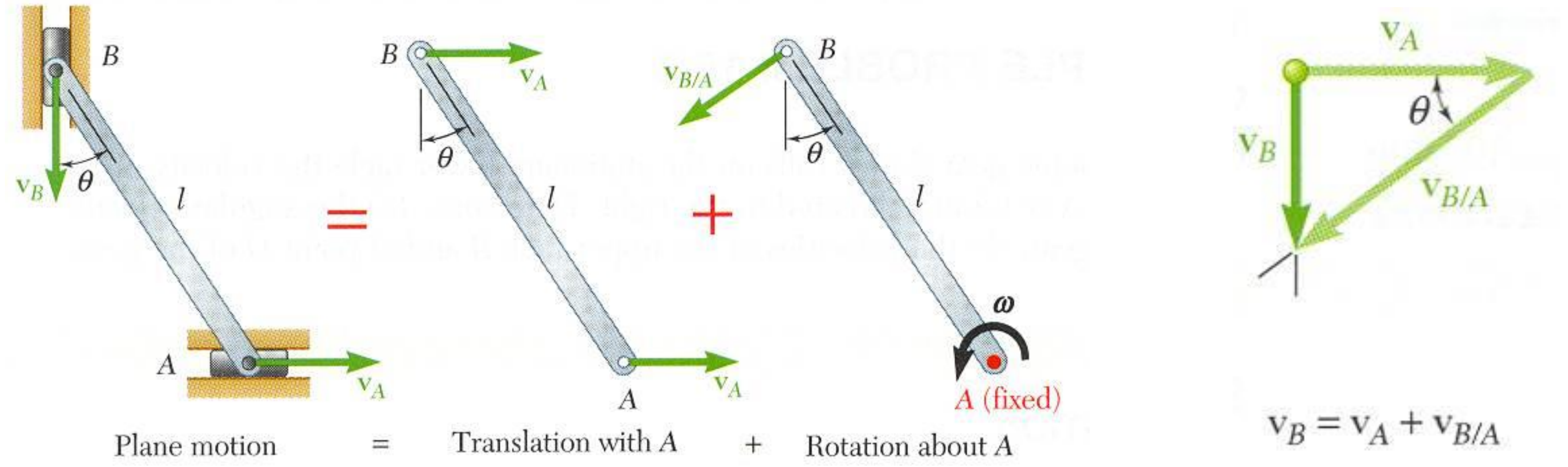
$$\omega = \frac{v_A}{l\cos\theta}$$

Absolute and Relative Velocity in Plane Motion



- Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. Angular velocity is not dependent on the choice of reference point.

Absolute and Relative Velocity in Plane Motion



- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.
- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

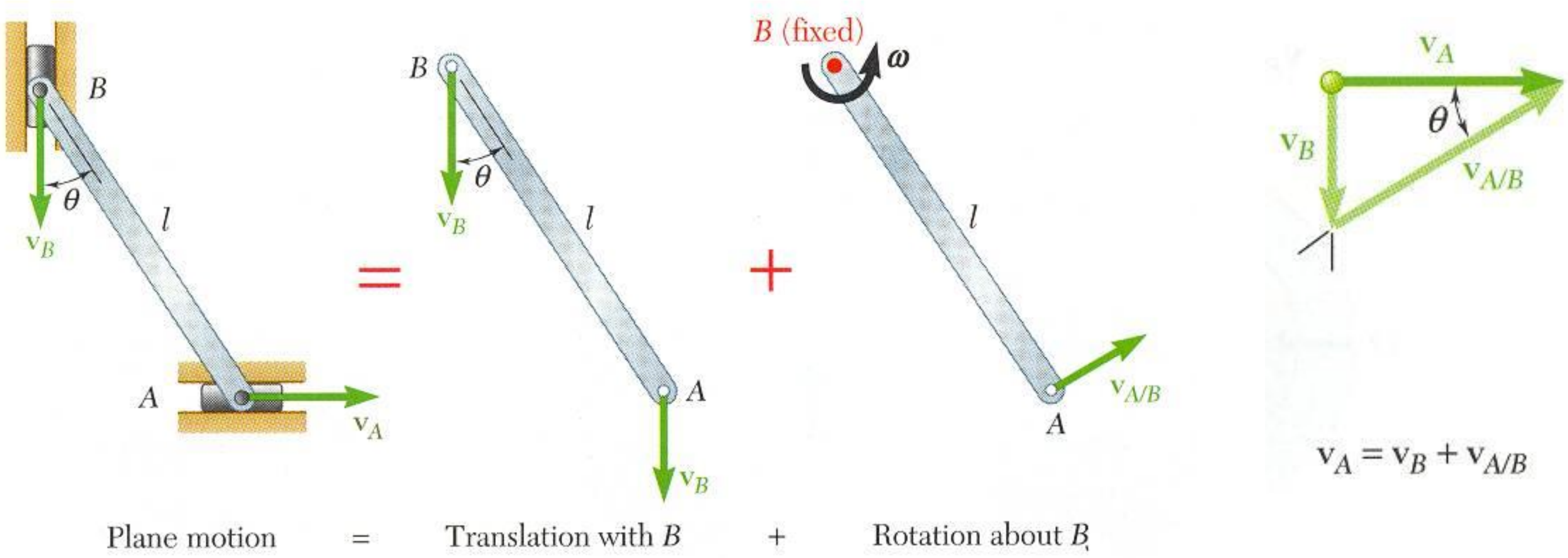
$$\frac{v_B}{v_A} = \tan\theta$$

$$v_B = v_A \tan\theta$$

$$\frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

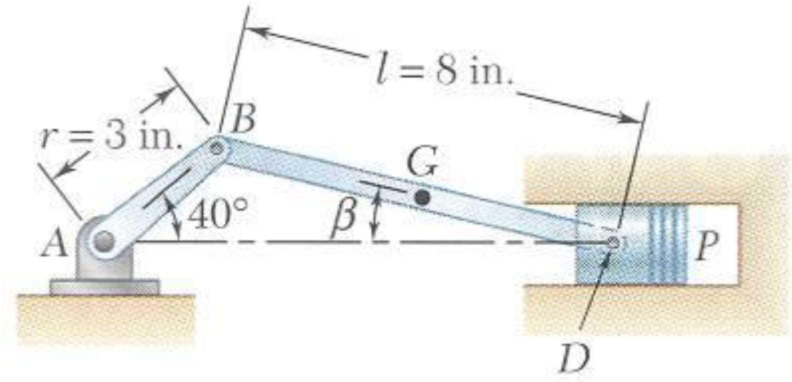
$$\omega = \frac{v_A}{l\cos\theta}$$

Absolute and Relative Velocity in Plane Motion



- Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. Angular velocity is not dependent on the choice of reference point.

Sample Problem 15.3



The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

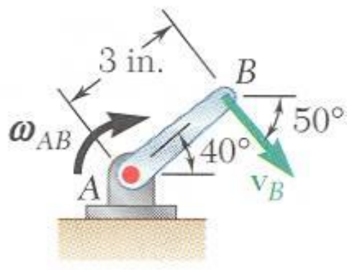
<u>SOLUTION</u>:

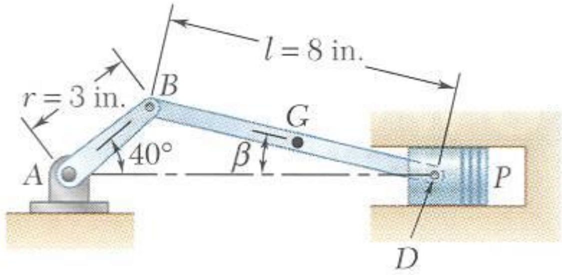
- Will determine the absolute velocity of point *D* with

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

- The velocity $\vec{v}_B$ is obtained from the given crank rotation data.
- The directions of the absolute velocity $\vec{v}_D$ and the relative velocity $\vec{v}_{D/B}$ are determined from the problem geometry.
- The unknowns in the vector expression are the velocity magnitudes v_D and $v_{D/B}$ which may be determined from the corresponding vector triangle.
- The angular velocity of the connecting rod is calculated from $v_{D/B}$.

Sample Problem 15.3





<u>SOLUTION</u>:

- Will determine the absolute velocity of point *D* with

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

- The velocity $\vec{v}_B$ is obtained from the crank rotation data.

$$\omega_{AB} = \left(2000\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{\text{min}}{60\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right) = 209.4\ \text{rad/s}$$

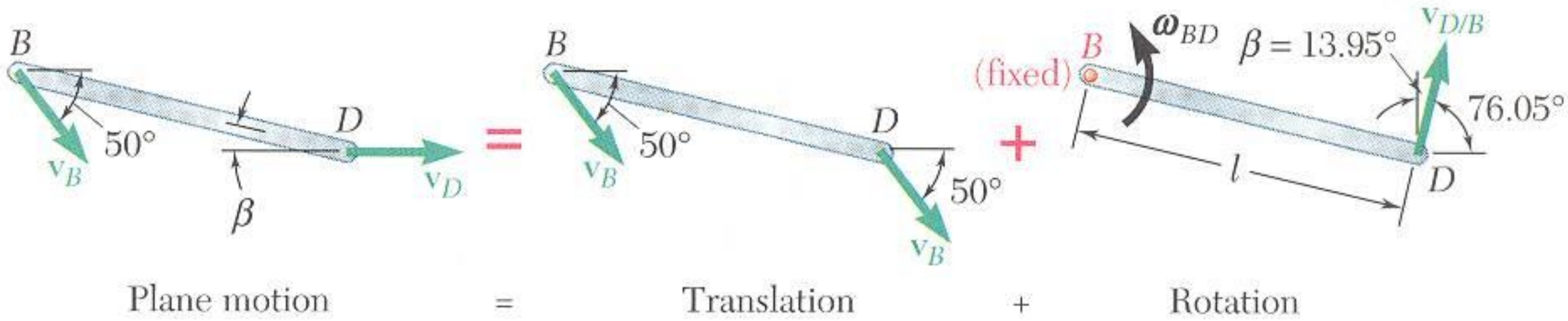
$$v_B = (AB)\omega_{AB} = (3\,\text{in.})(209.4\ \text{rad/s})$$

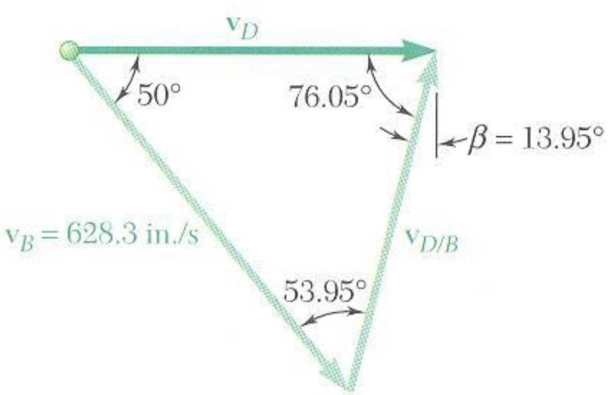
The velocity direction is as shown.

- The direction of the absolute velocity $\vec{v}_D$ is horizontal. The direction of the relative velocity $\vec{v}_{D/B}$ is perpendicular to *BD*. Compute the angle between the horizontal and the connecting rod from the law of sines.

$$\frac{\sin 40^\circ}{8\,\text{in.}} = \frac{\sin\beta}{3\,\text{in.}} \qquad \beta = 13.95^\circ$$

Sample Problem 15.3





$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

- Determine the velocity magnitudes v_D and $v_{D/B}$ from the vector triangle.

$$\frac{v_D}{\sin 53.95°} = \frac{v_{D/B}}{\sin 50°} = \frac{628.3\,\text{in./s}}{\sin 76.05°}$$

$$v_D = 523.4\,\text{in./s} = 43.6\,\text{ft/s}$$

$$v_P = v_D = 43.6\,\text{ft/s}$$

$$v_{D/B} = 495.9\,\text{in./s}$$

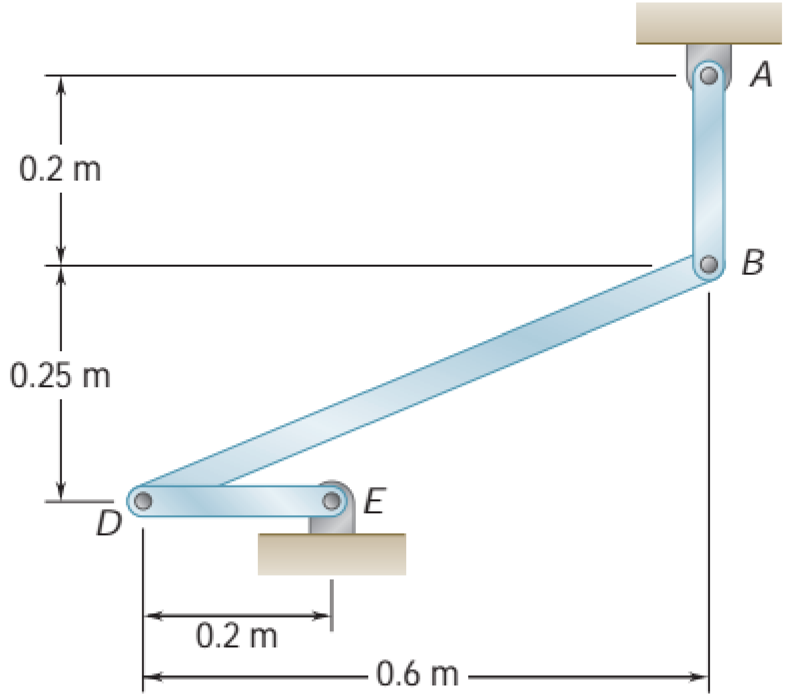
$$v_{D/B} = l\omega_{BD}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} = \frac{495.9\,\text{in./s}}{8\,\text{in.}}$$
$$= 62.0\,\text{rad/s}$$

$$\vec{\omega}_{BD} = (62.0\,\text{rad/s})\vec{k}$$

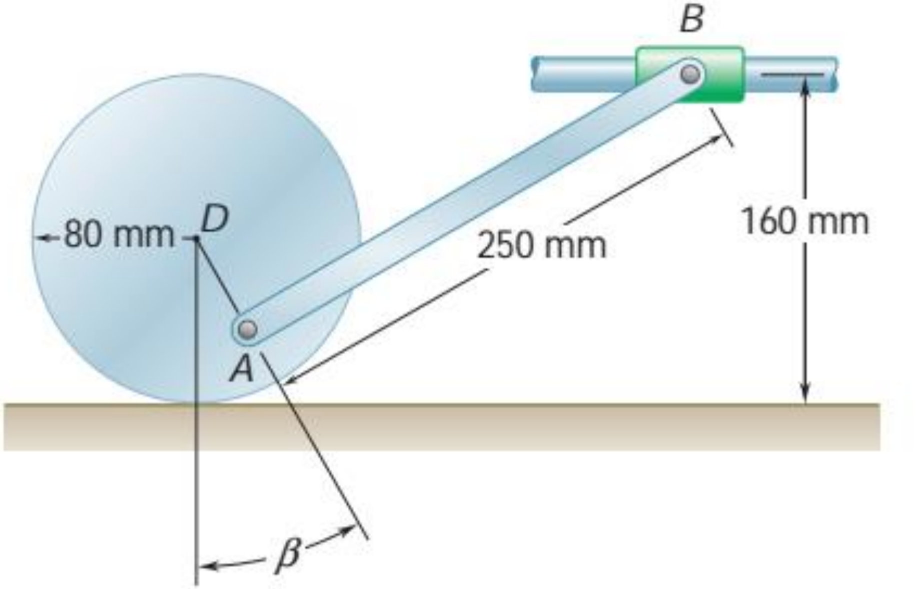
Prob # 15.63

Knowing that at the instant shown the angular velocity of rod *AB* is 15 rad/s clockwise, determine
(*a*) the angular velocity of rod *BD*,
(*b*) the velocity of the midpoint of rod *BD*.

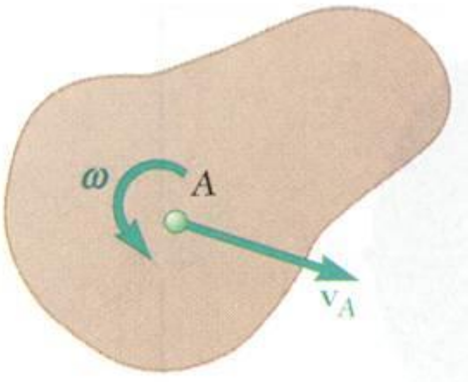


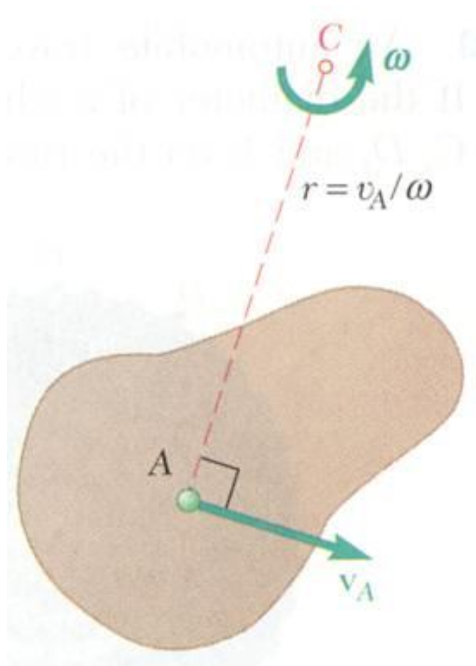
Prob# 15.71

The 80-mm-radius wheel shown rolls to the left with a velocity of 900 mm/s. Knowing that the distance *AD* is 50 mm, determine the velocity of the collar and the angular velocity of rod *AB* when (*a*) b = 0, (*b*) b = 90° .



Instantaneous Center of Rotation in Plane Motion





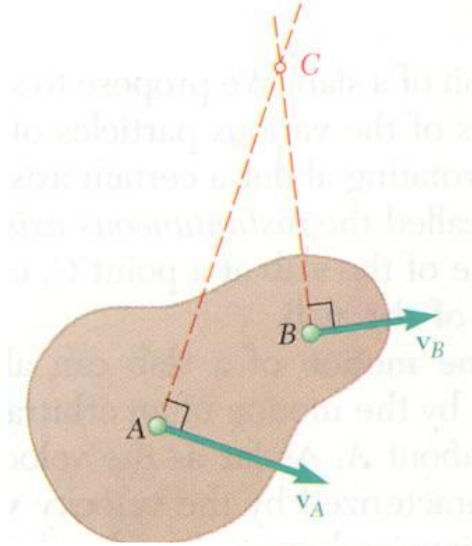
- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.

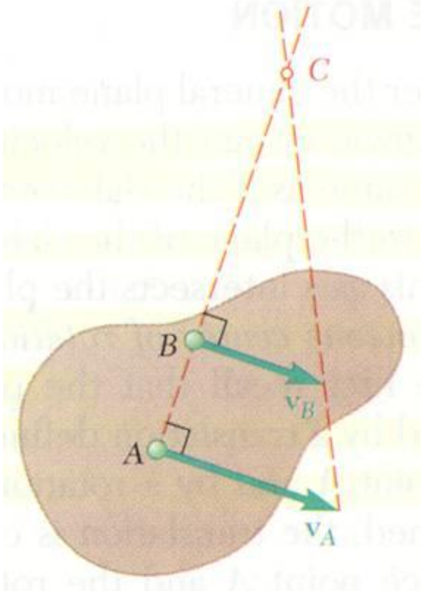
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.

- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

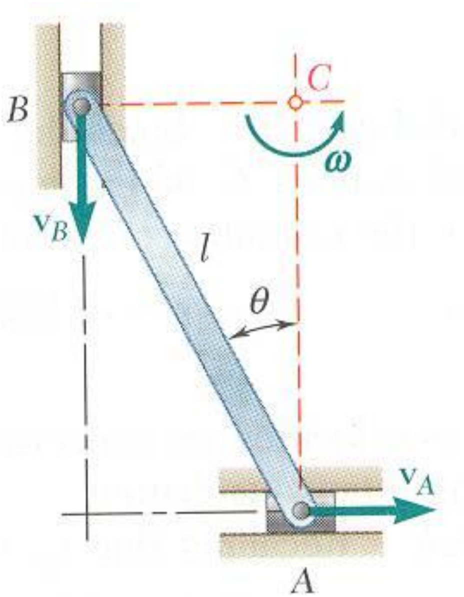
Instantaneous Center of Rotation in Plane Motion





- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .
- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.
- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.
- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

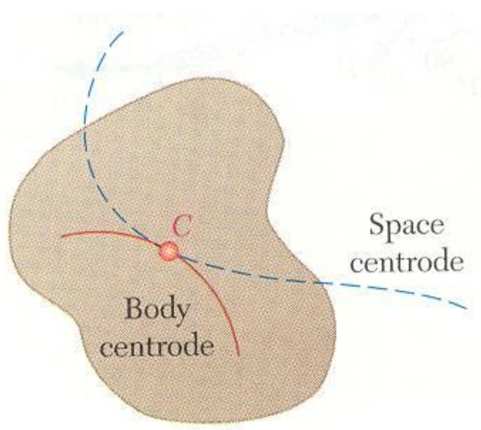
Instantaneous Center of Rotation in Plane Motion



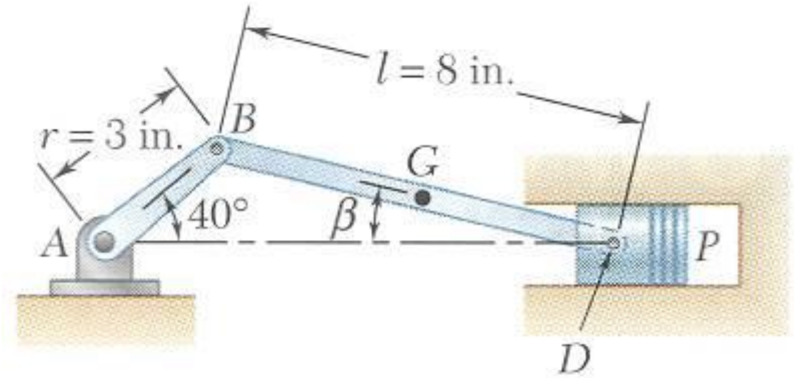
- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through A and B .

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta} \qquad v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta}$$
$$= v_A \tan\theta$$

- The velocities of all particles on the rod are as if they were rotated about C.
- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of rotation is not zero.
- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about C.
- The trace of the locus of the center of rotation on the body is the body centrode and in space is the space centrode.



Sample Problem 15.5



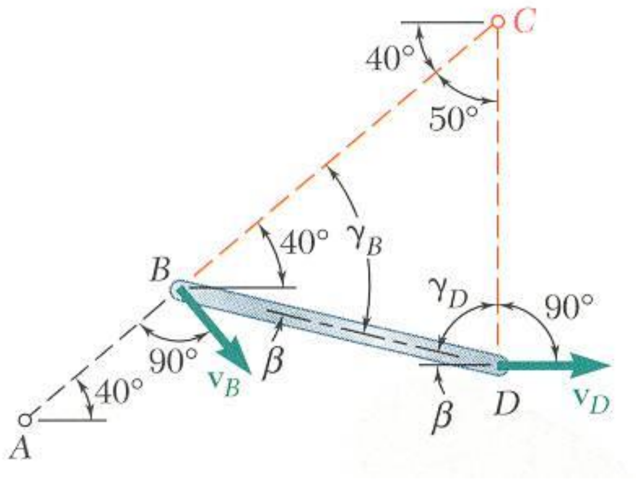
The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

<u>SOLUTION</u>:

- Determine the velocity at *B* from the given crank rotation data.
- The direction of the velocity vectors at *B* and *D* are known. The instantaneous center of rotation is at the intersection of the perpendiculars to the velocities through *B* and *D*.
- Determine the angular velocity about the center of rotation based on the velocity at *B*.
- Calculate the velocity at *D* based on its rotation about the instantaneous center of rotation.

Sample Problem 15.5



$$\gamma_B = 40° + \beta = 53.95°$$
$$\gamma_D = 90° - \beta = 76.05°$$

$$\frac{BC}{\sin 76.05°} = \frac{CD}{\sin 53.95°} = \frac{8\text{ in.}}{\sin 50°}$$

$$BC = 10.14\text{ in.} \quad CD = 8.44\text{ in.}$$

<u>SOLUTION</u>:

- From Sample Problem 15.3,

$$\vec{v}_B = (403.9\vec{i} - 481.3\vec{j})(\text{in./s}) \qquad v_B = 628.3\text{in./s}$$
$$\beta = 13.95°$$

- The instantaneous center of rotation is at the intersection of the perpendiculars to the velocities through *B* and *D*.

- Determine the angular velocity about the center of rotation based on the velocity at *B*.

$$v_B = (BC)\omega_{BD}$$
$$\omega_{BD} = \frac{v_B}{BC} = \frac{628.3\text{in./s}}{10.14\text{ in.}}$$

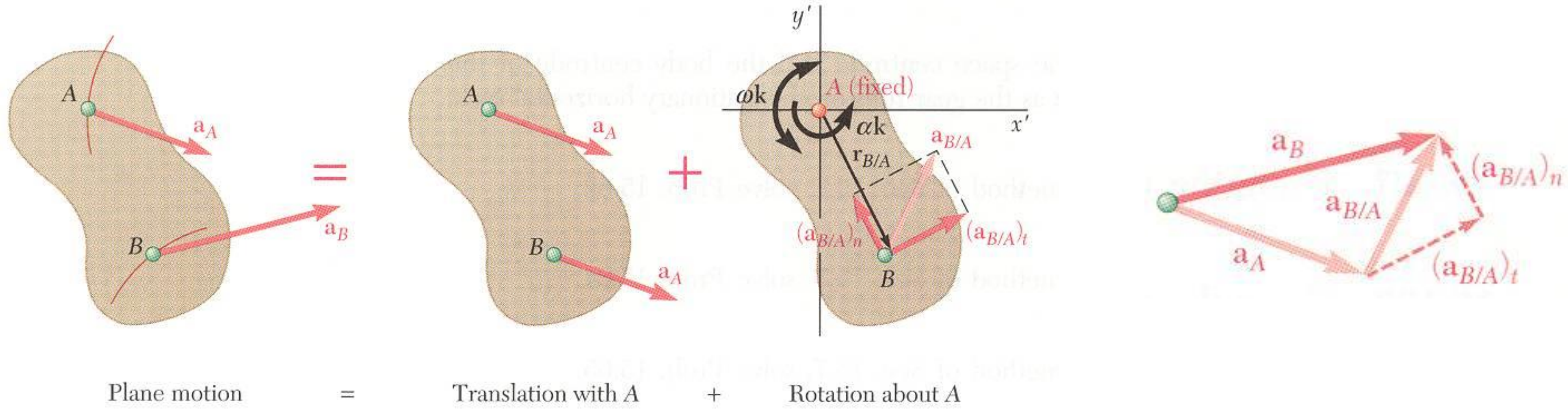
$$\boxed{\omega_{BD} = 62.0\,\text{rad/s}}$$

- Calculate the velocity at *D* based on its rotation about the instantaneous center of rotation.

$$v_D = (CD)\omega_{BD} = (8.44\text{ in.})(62.0\,\text{rad/s})$$

$$\boxed{v_P = v_D = 523\,\text{in./s} = 43.6\,\text{ft/s}}$$

Absolute and Relative Acceleration in Plane Motion



- Absolute acceleration of a particle of the slab,

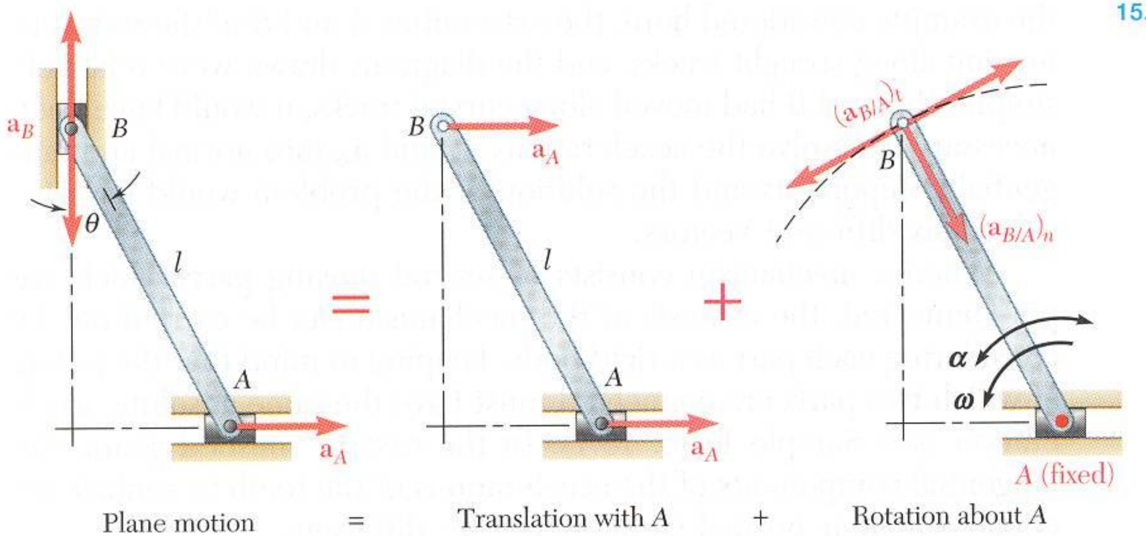
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about A includes tangential and normal components,

$$\left(\vec{a}_{B/A}\right)_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_t = r\alpha$$

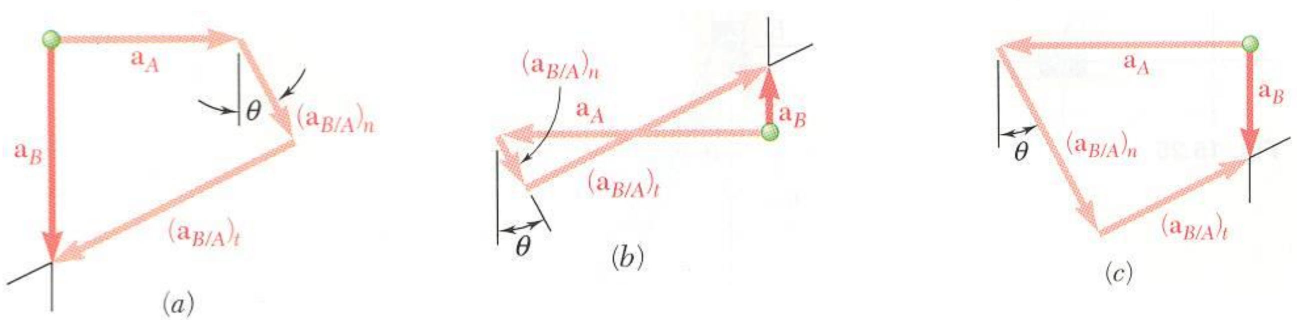
$$\left(\vec{a}_{B/A}\right)_n = -\omega^2 \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_n = r\omega^2$$

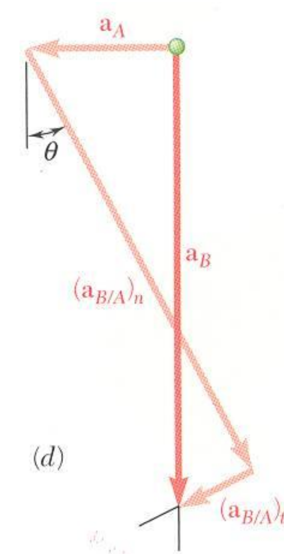
Absolute and Relative Acceleration in Plane Motion



- Given $\vec{a}_A$ and $\vec{v}_A$, determine $\vec{a}_B$ and $\vec{\alpha}$.

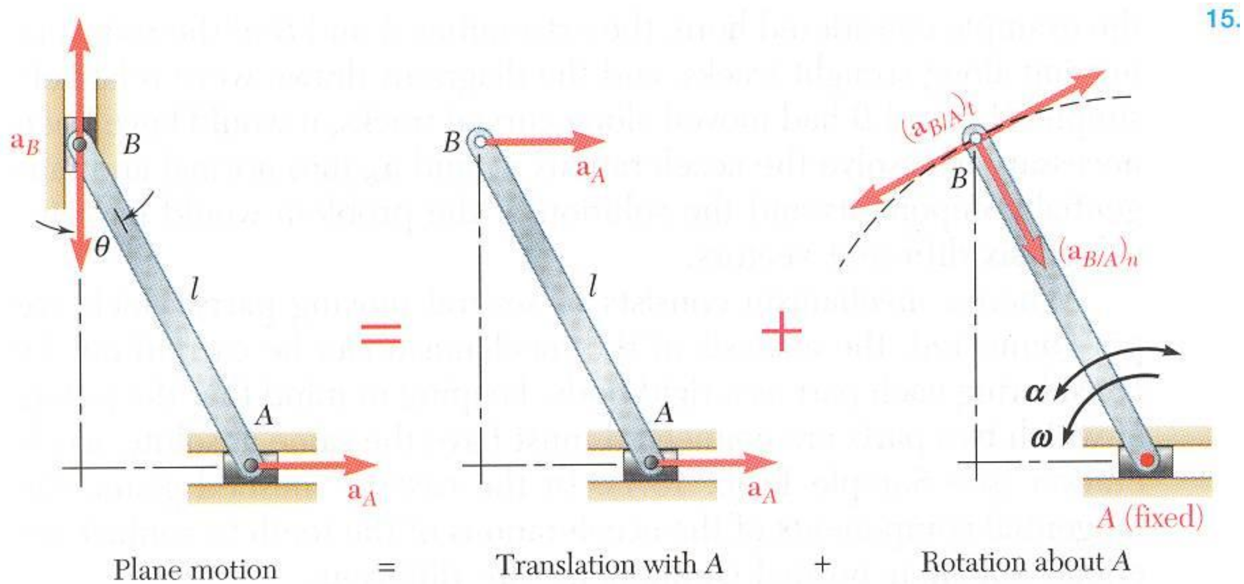
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_n + \left(\vec{a}_{B/A}\right)_t$$

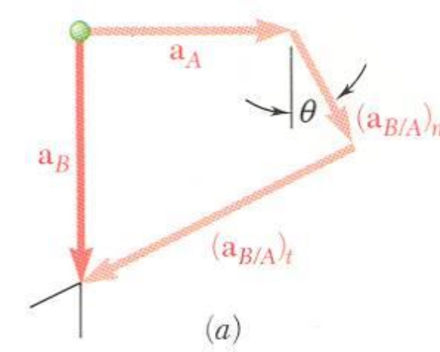




- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $\left(a_{B/A}\right)_n$
- Must also know angular velocity ω.

Absolute and Relative Acceleration in Plane Motion





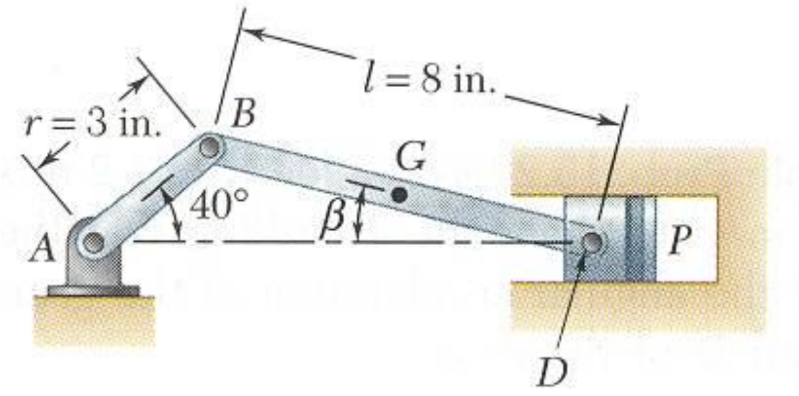
- Write $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ in terms of the two component equations,

$\overset{+}{\rightarrow}$ x components: $$0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$$

$+\uparrow$ y components: $$-a_B = -l\omega^2 \cos\theta - l\alpha\sin\theta$$

- Solve for a_B and α.

Sample Problem 15.7



Crank *AG* of the engine system has a constant clockwise angular velocity of 2000 rpm.

For the crank position shown, determine the angular acceleration of the connecting rod *BD* and the acceleration of point *D*.

SOLUTION:

- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from

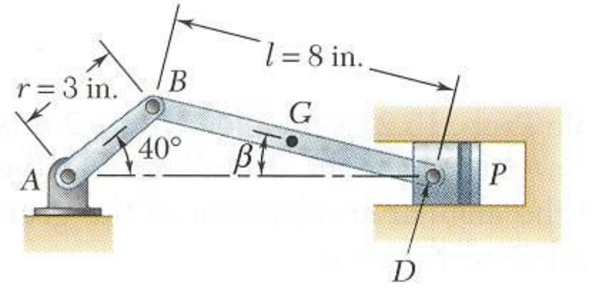
  $$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$

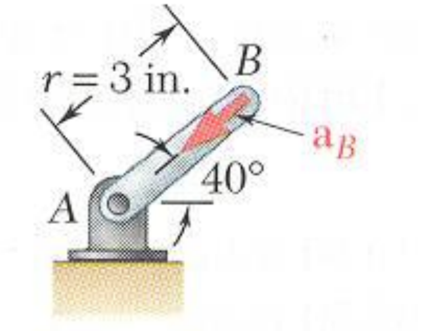
- The acceleration of *B* is determined from the given rotation speed of *AB*.

- The directions of the accelerations $\vec{a}_D, \left(\vec{a}_{D/B}\right)_t$, and $\left(\vec{a}_{D/B}\right)_n$ are determined from the geometry.

- Component equations for acceleration of point *D* are solved simultaneously for acceleration of *D* and angular acceleration of the connecting rod.

Sample Problem 15.7





SOLUTION:

- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$

- The acceleration of *B* is determined from the given rotation speed of *AB*.

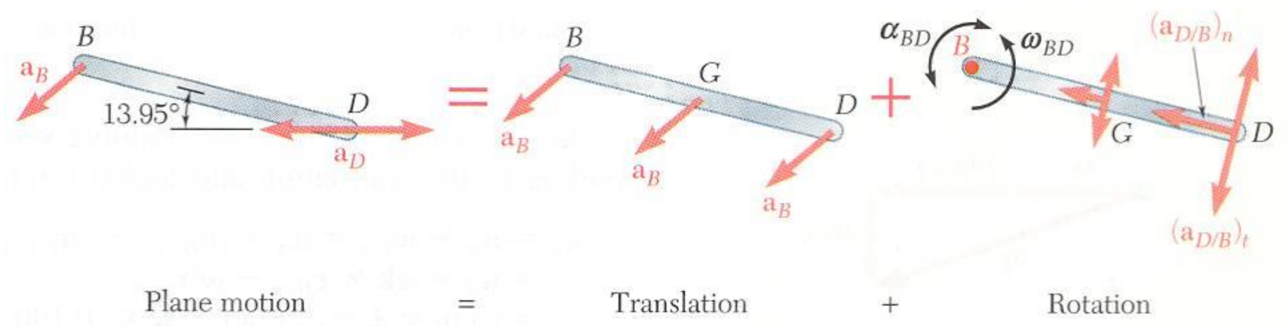
$$\omega_{AB} = 2000\ \text{rpm} = 209.4\ \text{rad/s} = \text{constant}$$

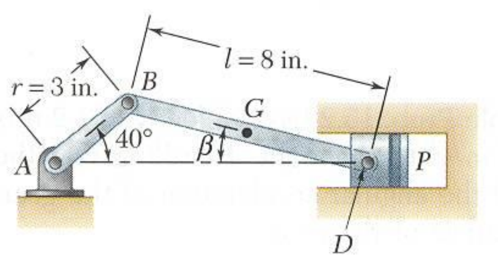
$$\alpha_{AB} = 0$$

$$a_B = r\omega_{AB}^2 = \left(\tfrac{3}{12}\ \text{ft}\right)\left(209.4\ \text{rad/s}\right)^2 = 10{,}962\ \text{ft/s}^2$$

$$\vec{a}_B = \left(10{,}962\ \text{ft/s}^2\right)\left(-\cos 40°\,\vec{i} - \sin 40°\,\vec{j}\right)$$

Sample Problem 15.7





- The directions of the accelerations $\vec{a}_D$, $(\vec{a}_{D/B})_t$, and $(\vec{a}_{D/B})_n$ are determined from the geometry.

$$\vec{a}_D = \mp a_D \vec{i}$$

From Sample Problem 15.3, ω_{BD} = 62.0 rad/s, β = 13.95°.

$$(a_{D/B})_n = (BD)\omega_{BD}^2 = \left(\tfrac{8}{12}\text{ ft}\right)(62.0\,\text{rad/s})^2 = 2563\,\text{ft/s}^2$$

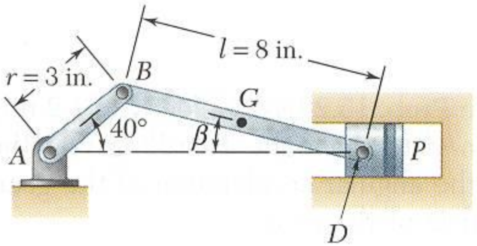
$$(\vec{a}_{D/B})_n = \left(2563\,\text{ft/s}^2\right)\left(-\cos 13.95°\vec{i} + \sin 13.95°\vec{j}\right)$$

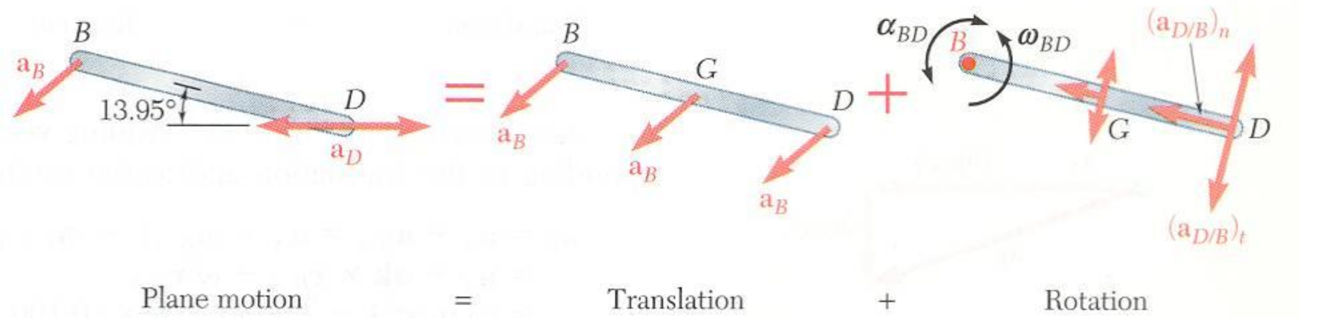
$$(a_{D/B})_t = (BD)\alpha_{BD} = \left(\tfrac{8}{12}\text{ ft}\right)\alpha_{BD} = 0.667\alpha_{BD}$$

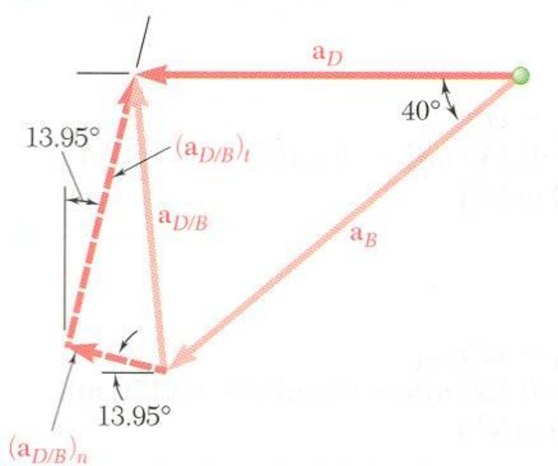
The direction of $(a_{D/B})_t$ is known but the sense is not known,

$$(\vec{a}_{D/B})_t = (0.667\alpha_{BD})\left(\pm\sin 76.05°\vec{i} \pm \cos 76.05°\vec{j}\right)$$

Sample Problem 15.7







- Component equations for acceleration of point D are solved simultaneously.

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$

x components:

$$-a_D = -10{,}962\cos 40^\circ - 2563\cos 13.95^\circ + 0.667\alpha_{BD}\sin 13.95^\circ$$

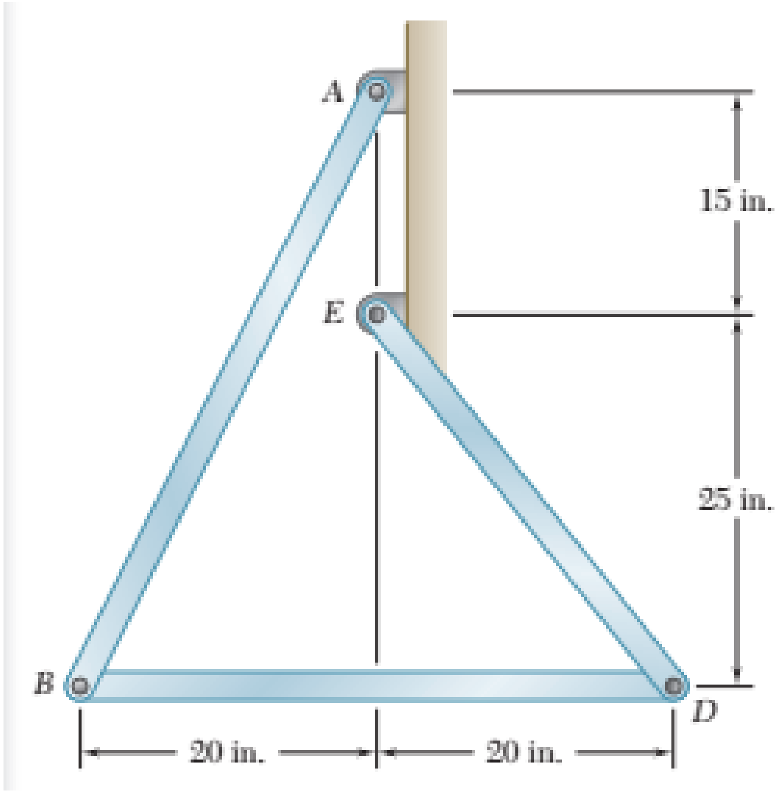
y components:

$$0 = -10{,}962\sin 40^\circ + 2563\sin 13.95^\circ + 0.667\alpha_{BD}\cos 13.95^\circ$$

$$\vec{\alpha}_{BD} = \left(9940\ \mathrm{rad/s^2}\right)\vec{k}$$
$$\vec{a}_D = -\left(9290\ \mathrm{ft/s^2}\right)\vec{i}$$

Prob # 15.131

Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration (*a*) of bar *BD*, (*b*) of bar *DE.*



Prob # 15.123

The disk shown has a constant angular velocity of 500 rpm counterclockwise. Knowing that rod *BD* is 10 in. long, determine the acceleration of collar *D* when (*a*) $\theta = 90°$, (*b*) $\theta = 180°$.